GLOBAL ILLUMINATION IN INHOMOGENEOUS MEDIA BASED ON CURVED PHOTON MAPPING

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Abstract

This paper deals with representing several of the phenomena that occur when light travels a curved path through inhomogeneous media, such as the well-known mirages. Traditional curved ray tracing presents several problems which have not been efficiently solved yet. We present here our approach based both on curved ray tracing and photon mapping, introducing the concept of curved photon mapping.

Keywords: Rendering, global illumination, curved ray tracing, photon mapping, natural phenomena, inhomogeneous media

1 Introduction and state of the art

Most ray tracing algorithms are based on the assumption that light rays travel following a straight path. This is so because said algorithms consider only homogeneous media, with properties being constant. While this assumption works well for a great number of situations, it nevertheless imposes several restrictions on the scenes and effects that can be reproduced, with probably mirages being the most notorious. Most of the media, however, are in fact inhomogeneous to one degree or another, with properties varying continuously from point to point. The atmosphere, for instance, is in fact inhomogeneous, since pressure, temperature and other properties do vary from point to point, and therefore its optic characterization, given by the index of refraction, is not constant.

A light ray propagating in a straight line would only be accurate in two situations: either there is no media through which the light travels (as in outer space, for instance), or the media are homogeneous. But with inhomogeneous media, new phenomena occur. For instance, light in warm air (which has a lower optical density) moves faster than in cold air, which is denser. According to Fermat's Principle, light crossing a medium gets curved towards the areas with the greater index of refraction. This index of refraction, which defines the optical characteristics of the medium, is a function of both humidity and density, as well as wavelength, with density being a function of pressure and temperature itself. Therefore, in a medium where temperature changes continually, so will the direction of propagation, thus making light rays travel curved paths (with the degree of curvature a function of wavelength).

Several of the atmospheric effects we see in nature, from mirages to the green flash, are owed to light traveling curved paths [1], and therefore are impossible or exceedingly costly to simulate with synthetic imagery using standard Monte Carlo ray tracing techniques. Nevertheless, modeling of nature has been one of the most ambitious goals of the Computer Graphics community.

There are several examples that simulate the interaction of light in the atmosphere, such as the works of Musgrave [2] or Nishita [3]. There is also some previous work on curved ray tracing in inhomogeneous media. Berger and Trout [4] recreate mirages by subdividing the medium into various homogeneous layers, with a different index of refraction for each one. Musgrave [5] proposes a purely reflective model as the means of forming mirages, while Groeller [6] uses sources of nonlinearity such as gravity centers, gravity lines, chaotic systems and parametric curved rays. Stam and Languenou [7] propose a solution by obtaining the differential equation that describes the trajectory of the ray from the equation of the light wave. Finally, Serón et al. [8] describe a more general method, free of the restrictions that appear in the above papers, regarding the dependences of the index of refraction, and propose a partial solution to the problem using the general equation, based on Fermat's principle, that describes the phenomenon. None of these works, though, can successfully follow the complete light paths, from the lights through inhomogeneous media to interaction with geometry, through inhomogeneous media again before finally reaching the eye. The basic problem of following all these paths is explained in the next section.

This paper describes a global illumination solution where the authors tackle one of the basic problems of curved ray tracing by combining it with photon mapping techniques. The behavior of light in inhomogeneous media is described in Section 2. Section 3 presents the curved photon mapping, with the results shown in Section 4. Conclusions and future work are presented in Section 5. Finally, the acknowledgements and bibliography appear in Sections 6 and 7 respectively.

2 Light in inhomogeneous media

Ray tracing is a well-documented technique for generating synthetic imagery [9], and the nuts and bolts of a ray tracer can be learnt elsewhere. Basically, in backward ray tracing, a ray is shot from the eye into the scene until it reaches an object, and from that intersection point more rays are shot towards the lights to find the color of the corresponding pixel.

This works well for homogeneous media, where the light rays travel in a straight path. But the situation changes when using curved ray tracing: curving the eye ray until it reaches an object in the scene is conceptually simple. We do not know where each ray is going to end up after being curved by the medium, but we do not care either, since it will eventually hit some object or be lost in infinity, just as with straight ray tracing. The problem arises when shooting additional rays from the intersection point to the lights. Even though we know both the start and end points of the ray, it is difficult to find a curve that passes through those two points while also obeying the nonlinearity caused by the inhomogeneous medium. Just shooting rays hoping one will eventually hit the light is obviously very ineffective. This is a problem similar to computing caustics by using path tracing or any other backward ray tracing method. Caustics are concentrated light reflections on diffuse surfaces caused by refraction through transparent surfaces, and to simulate them in the image it would be necessary to trace a random ray from a diffuse surface, and have the ray interact with several specular surfaces and then hit the light. Arvo [10] first introduced forward ray tracing to simulate caustics, while bidirectional path tracing [11] [12] combines both backward and forward ray tracing by generating some path vertices starting from the light sources and some from the eye.

However, all these techniques would fail again (or be too computationally expensive) in an inhomogeneous media. Shooting rays from the objects to the eye in forward ray tracing, or obtaining shadow rays in bidirectional path tracing would face the same basic problem as with backward ray tracing: to find a curve that passes through two given points while obeying the nonlinearities of the medium.

Groeller proposes several solutions to this problem in [6]: the first one supposes that the light rays are only curved from the eye to the intersection point, but travel in a straight line from the intersection point to the light source. The second solution consists in assigning color to the intersection point regardless of the light sources, for instance by using textures with the illumination already pre-calculated. These are obvious simplifications that will work well for general image synthesis, but do not reflect the physics of the system. The second one is in fact used in [8], since the authors concentrated mainly on solving the general equation accurately. Another idea is to voxelize the space and get the approximate direction of incoming light by shooting light rays through the voxels from each light, saving the results in a pre-computed structure. As far as the authors know, there is not much more literature regarding this problem.

As a starting point to implement our curved ray tracer, we take Fermat's principle [13], which can be formulated as "light, in going between two points, traverses the route *l* having the smallest optical path length *L*". The optical path *L* is defined as the index of refraction times the traveled path. In its differential form, it can be written as $dL=ndl$. According to Fermat's principle, the optical path along a light ray trajectory must be a minimum, therefore dL =0, where dL is given by:

$$dL = d\int_A^B ndl = \int_A^B \mathrm{dn}\,\mathrm{d}l + \int_A^B nd(dl) = \int_A^B \frac{\partial n}{\partial x_i} dx_i dl + \int_A^B nd(dl)$$

(Eq. 2.1)

where x_i are the components of *l*. Given that $\mathrm{dl}^2 = \mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2$, considering $\mathrm{dx_i}$ as variables and taking increments we get:

$$\delta(\mathrm{dl}) = \frac{\mathrm{dx_i}}{\mathrm{dl}}\delta(\mathrm{dx_i})$$

so that equation 2.1 results:

$$\int_A^B n\delta(\mathrm{dl}) = n\frac{\mathrm{dx_i}}{\mathrm{dl}}\delta x_i\Big]_A^B - \int_A^B \delta x_i \frac{d}{\mathrm{dl}}\left(n\frac{\mathrm{dx_i}}{\mathrm{dl}}\right)\mathrm{dl}$$

(Eq. 2.2)

Since the different considered trajectories start in the fixed points A and b, $dx_i(A)=0$ and $dx_i(B)=0$, so equation 2.2 results as follows:

$$\delta L = \int_A^B \left[\frac{\partial n}{\partial x_i} - \frac{d}{\mathrm{dl}}\left(n\frac{\mathrm{dx_i}}{\mathrm{dl}}\right)\right]\delta x_i \mathrm{dl} = 0$$

(Eq. 2.3)

This equation must be true for any value of δx_i, which lets us come up with the equation to obtain the trajectory of a light ray in an inhomogeneous medium with a known index of refraction, which is:

$$\frac{d}{dl}\left(n\frac{d\vec{r}}{dl}\right)-\nabla n=0\leftrightarrow\frac{d}{dl}\left(n\frac{dx_j}{dl}\right)-\frac{\P n}{\P x_j}=0$$

(j=1,2,3)

(Eq. 2.4)

where l is the length of the arc, n is the index of refraction of the medium and $\vec{r}=x_j$ with (j=1,2,3) are the coordinates of the point. If the index of refraction is known for every point of the medium, we first calculate that index and the slope of the curve at step *i*, advance Δl_i along the direction of the tangent to reach step *i+1*, and calculate the new index of refraction and tangent again. To calculate the direction of the tangent we first obtain a numerical approximation by discretizing the equation, effectively replacing differentials by increments. We then apply Euler's method to the equation and the Richardson's extrapolation algorithm to select an optimum integration step for each instant, given an estimate of the tolerable error. The process ends when we get to the intersection point of the path of the ray with an object.

3 Curved photon mapping

We have seen how curved ray tracing encounters a serious problem when shooting rays from the objects to the lights. The problem is inherent to backward ray tracing in inhomogeneous media. If we used forward ray tracing, curving rays from the lights to the objects would be feasible, but the symmetric problem would appear again when going from the objects to the eye. Since neither a pure backward nor forward ray tracing solution seems viable for inhomogeneous media where rays get curved, we sought a combined approach, based in photon mapping techniques.

Photon mapping is a two-pass algorithm [14]. In the first pass photons are shot from each light into the scene, and traced as they interact with the geometry. In each collision, the Russian roulette algorithm decides whether each photon is absorbed, reflected or transmitted. When the photon hits a diffuse material, it is stored. The data structure that stores all these photons is called the photon map. The second pass consists on a modified ray tracing algorithm, but instead of shooting shadow rays, radiance is estimated from the nearest stored photons around the intersection point. Specular reflections are obtained by standard ray tracing, since the probabilities of a photon being reflected in the exact specular direction towards the eye are infinitesimal.

On the other hand, the direct illumination is more accurately computed through ray tracing, by sending shadow rays towards the lights and testing for occlusions by other objects in the scene. As a consequence, the practical approach proposed by Wann Jensen in [11] uses two maps, one for caustics and the second one for indirect illumination only.

The photon mapping algorithm allows a full global illumination simulation, including color bleeding and caustic generation, neither of which can be obtained by traditional recursive ray tracing. Although Monte Carlo-based ray tracing techniques do handle them well, they present their own set of problems. Path tracing, for instance, is an unbiased algorithm capable of obtaining a complete solution, but the resulting images show artifacts in the form of high-frequency noise due to the random directions chosen for the paths of the rays. Moreover, its convergence is slow: to halve the error we need to compute four times as many samples. Photon-mapping-based images are on the contrary free of high frequency noise, and are in general more efficiently obtained than path tracing or radiosity. The trade-off is that the method is no longer unbiased, but it is nevertheless consistent.

Our solution to obtain a full global illumination solution in inhomogeneous media is based on exploiting the independency in the photon map algorithm between light propagation (photon casting and tracing, first pass) and visibility determination (ray tracing, second pass). This idea is implemented by the *curved photon-mapping algorithm*. It works almost the same way than the standard photon-mapping algorithm, but by using Fermat's principle and solving equation 2.4 for both photons and ray tracing, it can handle inhomogeneous media as well. It is then also a two-pass algorithm: first, the path of each emitted photon is curved as it is being traced throughout the scene, using equation 2.4. Storing the photons (which paths have now been curved) on diffuse surfaces follows the same process as though the photons had not been curved. Two maps are used in the current version of this technique: a caustics photon map, to represent caustics in a more efficient way using projection maps, and a second photon map to represent both direct and indirect illumination, excluding caustics. We have named this second map the diffuse photon map.

During the second pass, the rays traced from the eye are also curved using again equation 2.4. To account for direct illumination, no shadow rays are shot towards the lights as in the original photon mapping algorithm; instead, radiance is estimated by using the diffuse photon map. This way, we avoid having to find the curved shadow ray that links the intersection point and the light while obeying the nonlinearities of the medium.

4 Results

By combining curved ray tracing and curved photon mapping, we obtain a full global illumination algorithm that can reproduce effects like color bleeding or caustics in inhomogeneous media, solving the problem of having to shoot curved shadow rays. This is a step forward over traditional Monte Carlo or curved ray tracing, neither of which can simulate these effects, or at least not within reasonable computational limits.

In order to accelerate the computing of the photon map, a projection map has been used [14]. We, however, use a more conservative map. Instead of just casting rays from the lights through the center of each cell of the projection map and see if it hits any surface, rays are cast through the corners of the cells. The cell will be active if any of the rays cast through each of its corners hits any surfaces. Note that this is equivalent to using a map with a smaller grid.

The next figures show the results obtained so far. As our simple test scene we use a closed, square room with a cross-shaped window in the left wall. A strong spotlight is placed outside looking inside, so that most of its light enters the room through the cross-shaped widow. No volumetric effects are computed at this stage. Two different settings were modeled: the medium inside the room has a constant index of refraction in Figures 1 and 2, while it varies in Figures 3 and 4, thus curving the light rays.

Figure 1 shows the scene rendered with ray tracing and photon mapping in homogeneous media. Figure 2 shows the photon map computed for the former figure. In Figure 3 the media has become non-homogeneous, making the index of refraction inside the room a function of height and consequently curving the photon paths and the rays. The resulting curved photon map can be seen in Figure 4.

The images were rendered at a 400x300 resolution, shooting 3.000.000 photons from the spotlight and using 150 photons for estimating the irradiance. Figure 1 took 2’35’’ to render on a P-III @800Mhz and 512Gb of RAM, while Figure 3 took 4’47’’ on the same machine, owed to the added difficulty of calculating the curved paths. The error threshold when using Richardson’s extrapolation algorithm in this case was set to 0.01.

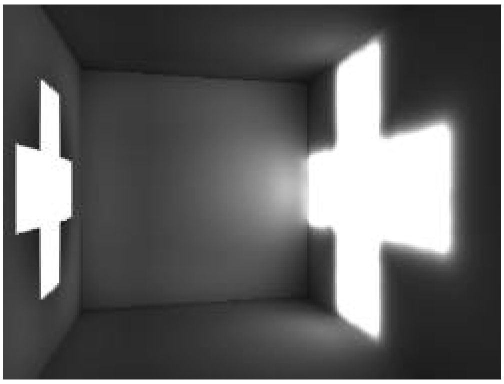

Figure 1. Image calculated using standard ray tracing and photon mapping in homogeneous media

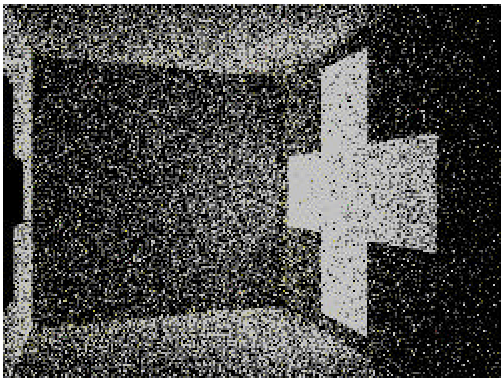

Figure 2. Photon map computed for the previous scene

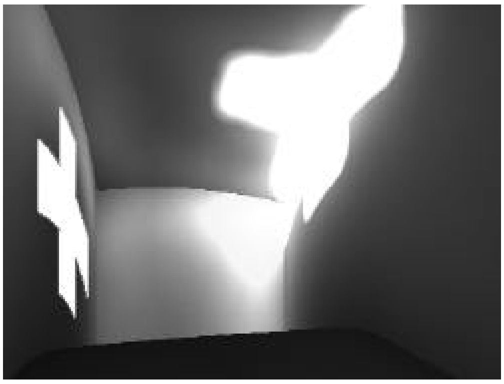

Figure 3. The same scene as in figure 1, but considering inhomogeneous media.

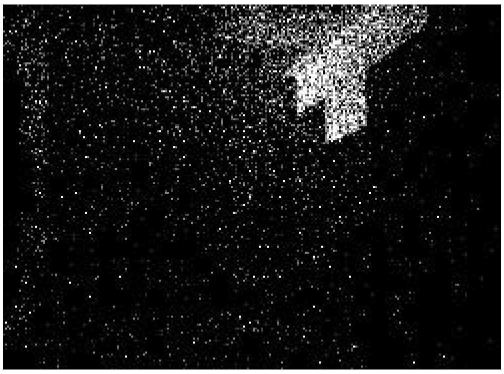

Figure 4. Resulting curved photon map from Figure 3.

5 Conclusions and future work

Curved photon mapping is a full global illumination algorithm that can handle homogeneous and inhomogeneous media. Both photons and rays are curved using Fermat's principle. To avoid having to find shadow rays, the direct illumination is computed from the diffuse photon map, which stores both direct and indirect diffuse hits.

The results obtained so far show the viability of the approach. As the system gets refined over time, we nevertheless plan to generate more complex images that mimic some natural phenomena impossible to simulate with standard ray tracing in homogeneous media. Regarding the curved ray tracer, we would like to use a more computationally efficient method to solve equation 2.4. Instead of using Euler's method, we plan to use a multipass method such as Adams-Basforth or Adams-Moulton [15].

We are also currently working with spectral data, instead of RGB values. The index of refraction *n* for inhomogeneous atmospheres is in fact a function of wavelength, which means that photons with a certain wavelength should get more curved than others. This, along with Rayleigh's dispersion, is for instance what causes the green flash effect, seen under very specific circumstances right before the last sun ray dies behind the horizon. Colored caustics in inhomogeneous media, as light gets refracted by a prism, will also be an interesting academic exercise. Cauchy's formula or Sellmeier's approximation are being used to obtain *n* as a function of wavelength.

Curved photon mapping seems also easy to integrate with volume photon maps and ray marching for subsurface scattering and other effects involving participating media, again computing direct illumination using the diffuse map. Possible areas of interest are underwater imagery or driving simulators where the effects of fog need to be accounted for while simulating visibility.

Finally, given the fact that finding all the curved paths of both photons and rays is computationally very expensive, optimization strategies are specially important. Our current system only supports projection maps, although other standard optimisation techniques such as irradiance caching or importance sampling are also being developed. Another line of future work to reduce the rendering times implies the parallelization of the code to be run on a Beowulf system.

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7 Bibliography

[1] M. G. J. Minnaert, *Light and color in the outdoors*, (Springer-Verlag, 1993).
[2] F. K. Musgrave, Methods for realistic landscape rendering, PhD. Thesis, Yale University, 1993
[3] T. Nishita, Light scattering models for the realistic rendering of natural scenes, *Proceedings of rendering Techniques '98*, 1998, 1-10.
[4] M. Berger, T. Trout, Ray tracing mirages, *IEEE Computer Graphics and Applications, 11(5)*, may 1990, 36-41.
[5] F. K. Musgrave, A note on ray tracing mirages, *IEEE Computer Graphics and Applications*, *10(6)*, 1990, 10-12.
[6] E. Groeller, Nonlinear ray tracing: visualizing strange worlds, *Visual Computer 11(5)*, Springer Verlag, 1995, 263-274
[7] J. Stam, E. Languenou, Ray tracing in non-constant media, *Proceedings of Rendering Techniques '96*, 1996, 225-234
[8] F.J. Serón, D. Gutiérrez, G. Gutiérrez, E. Cerezo, An implementation of a curved ray tracer for inhomogeneous atmospheres, submitted to *ACM Transactions on Graphics, 2002.*
[9] A. S. Glassner, *An introduction to ray tracing*, (Academic Press Inc., 1989).
[10] J. Arvo, "Backward ray tracing", in Developments in ray tracing, SIGGRAPH '86 Sseminar Notes, volume 12, 1986.
[11] E. P. Lafortune, Y. D. Willems, Bidirectional path tracing, in *Compugraphics '93*, 1993, 95-104.
[12] E. Veach, L. Guibas, Bidirectional estimators for light transport, in *Fifth Eurographics Workshop on Rendering*, 1994, 147-162.
[13] S. A. Glassner, *Principles of digital image synthesis*. (Morgan Kauffman Publishers, Inc. ISBN 1-55860-276-3. 1995).
[14] H. W. Jensen, *Realistic image synthesis using photon mapping* (AK Peters, ISBN 1-56881-147-0. 2001)
[15] R. L. Burden, J. Douglas Faires, *Numerical Analysis* (PWS, Boston, USA, 1995).